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When two exponents of a N-eq. are greater than two & the other is bound to be restricted on two.

$a^x + b^y = c^z$, where there is no common factor among a, b, c.

if x, y > 2 then z = 2 & so on.

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ABSTRACT

This is the extended version of the papers approved for publication in Aug-edition, Vol-4, issue 8 where discussing matter was only on one exponent of a N-eq. greater than two and the other two exponents were restricted on two. Now it will be discussed how two exponents exceed two and other is bound to be restricted on 2.

Keywords

Beal equation, N_d operation, N_s operation, NZ-equation, Zygote elements

I. Introduction

$\mathbf{a^x + b^y = c^z}$, where a, b, c, x, y, z all are of positive integers & x, y, z > 2. If there is no common factor among a, b, c there exists two kinds of equations

a) N-eq. where zygote elements are of positive integer and in this case only one exponent is possible to be raised beyond two.

b) NZ-eq where at least one zygote element is of pure irrational form ($q\sqrt{r}$). In this case two exponents are possible to be raised beyond two and the other is bound to be restricted on two.

If all the three exponents exceed two it is known as Beal equation. How it is formed and under what condition, it has already been discussed.

II. The cases where two exponents are greater than two.

The N-eq. $a^2 + b^2 = c^2$, $a < b < c$ can be defined as $(a_0^2 - b_0^2)^2 + (2a_0b_0)^2 = (a_0^2 + b_0^2)^2$, where a, b, c are N-elements & a_0, b_o are its zygote elements.
$(a_0^2 - b_0^2)$ & $(a_0^2 + b_0^2)$ are the corresponding zygote expressions or corresponding N-elements.

If the zygote elements are of positive integers, we can have the equation
$a^x + b^y = c^z$ where (a, b, c) is a prime set and if $x > 2$, $y = 2 = z$ or, if $y > 2$, $z = 2 = x$ or, if $z > 2$, $x = 2 = y$

If the zygote elements are of irrational nature i.e. in the form of $(p \pm q\sqrt{r})$, we have the N-eq. renamed as N-eq. of irrational zygote elements or simply NZ -equation.
$\Rightarrow \{(p + q\sqrt{r})^2 - (p - q\sqrt{r})^2\}^2 + \{2(p + q\sqrt{r})\,(p - q\sqrt{r})\}^2 = \{(p + q\sqrt{r})^2 + (p - q\sqrt{r})^2\}^2$

or, $(4pq\sqrt{r})^2 + \{2(p^2 - q^2r)\}^2 = \{2(p^2 + q^2r)\}^2$
or, $\{p)^2 - (q\sqrt{r})^2\}^2 + \{2p.q\sqrt{r}\}^2 = \{\{p)^2 + (q\sqrt{r})^2\}^2$

Here also like N-eq. the RH term of NZ-equation can produce even power by virtue of N_s operation in between mother expression & self and odd power by same N_s operation in between mother & its zygote elements. Similarly, corresponding LH term of NZ-eq. can produce power by virtue of N_d operation.

These two N_s & N_d operation cannot run simultaneously. Hence, only one element can produce power greater than two. But after N_d or N_s operations, the irrational element can produce power due to presence of $\sqrt{r}$ factor. Here, if $x, y > 2$ then $z = 2$ & so on.
Let us write the N-eq. in power form:

$$(\alpha^2 - \beta^2)^n + \{\,{}^nc_1\alpha^{n-1}\beta + {}^nc_3\alpha^{n-3}\beta^3 + \ldots\ )^2 = \{\alpha^n + {}^nc_2\alpha^{n-2}\beta^2 + \ldots\}^2 \qquad \ldots\ldots(A)$$

$$\{\alpha^n - {}^nc_2\alpha^{n-2}\beta^2 + \ldots\}^2 + \{\,{}^nc_1\alpha^{n-1}\beta - {}^nc_3\alpha^{n-3}\beta^3 + \ldots\ \}^2 = (\alpha^2 + \beta^2)^n \qquad \ldots\ldots(B)$$

For NZ-eq. where α is integer & β is irrational Eq.(A) can be written in two ways.

$(\alpha^2 - \beta^2)^n + \{\beta f(\alpha, \beta, n)\}^2 = \{\alpha g(\alpha, \beta, n)\}^2$ when n is odd. ……. (A1)
$(\alpha^2 - \beta^2)^n + \{\alpha\beta f(\alpha, \beta, n)\}^2 = \{g(\alpha, \beta, n)\}^2$ when n is even. ……. (A2)

The integer element i.e. third one cannot produce power. If the irrational element i.e. second one produces power, $f(\alpha, \beta, n)$ must be in the form of β^{2m} in case of n is odd and in the form of $(\alpha\beta)^{2m}$ in case of n is even.

Similarly Eq.(B) can be written in two ways

$\{\alpha g_1(\alpha, \beta, n)\}^2 + \{\beta f_1(\alpha, \beta, n)\}^2 = (\alpha^2 + \beta^2)^n$ when n is odd ……(B1)
$\{g_1(\alpha, \beta, n)\}^2 + \{\alpha\beta f_1(\alpha, \beta, n)\}^2 = (\alpha^2 + \beta^2)^n$ when n is even ……(B2)

The integer element i.e. first one cannot produce power. If the irrational element i.e. second one produces power, $f_1(\alpha, \beta, n)$ must be in the form of β^{2m} in case of n is odd and in the form of $(\alpha\beta)^{2m}$ in case of n is even.

Example in favor of Eq.(B1)

for n = 3, $3\alpha^2 - \beta^2 = \beta^m$ or, $\beta^m + \beta^2 - 3\alpha^2 = 0$, where obviously, m is even & β is in the form of $q\sqrt{r}$, (q, r are odd) & there is no c.f. among α, p, q.
We have, $(\sqrt{3})^4 + (\sqrt{3})^2 = 3.2^2$ Hence, consider the equation,
$\{2^2 - (\sqrt{3})^2\}^2 + (2.2\sqrt{3})^2 = \{2^2 + (\sqrt{3})^2\}^2$ i.e. $1^2 + (4\sqrt{3})^2 = 7^2$.
By N_S operation in between 7^2 & 7 i.e. in between $1^2 + (4\sqrt{3})^2$ & $2^2 + (\sqrt{3})^2$ we get, $(8\sqrt{3} \pm \sqrt{3})^2 + (2 \mp 12)^2$ where one case is $3^5 + 10^2 = 7^3$.

Example in favor of Eq.(B2)

For n = 4, we have the irrational element $\beta.4(\beta^2 - \alpha^2)$. Put $\alpha = 1$ & $\beta = \sqrt{2}$, we get$4(\beta^2 - \alpha^2) = 4 = (\sqrt{2})^4 = \beta^4$
Hence, consider the equation $\{(\sqrt{2})^2 - 1^2\}^2 + (2\sqrt{2})^2 = \{(\sqrt{2})^2 + 1^2\}^2$
or, $1^2 + (2\sqrt{2})^2 = 3^2$. Apply N_s operations in between $\{1^2 + (2\sqrt{2})^2\}$ & self.
$(2\sqrt{2} \pm 2\sqrt{2})^2 + (8 \mp 1)^2 = 3^2.3^2$ or, $2^5 + 7^2 = 3^4$ or, directly from Eq-(B) we get the same result.
On the same logic for n = 2, we get $1 + 2^3 = 3^2$.

As both the binomially expanded elements under Eq-(B) are sum of alternately (+) & (–), it will produce the relations of low value elements. But Eq-(A) will produce relations of high value elements.

Let us take the example of $17^3 + 2^7 = 71^2$. It is an example of low value elements. Here, if we proceed from 71 for N_s operation , by back calculation we can say,
$71 = [\sqrt{\{(71 + 8\sqrt{2})/2\}}]^2 + [\sqrt{\{(71 - 8\sqrt{2})/2\}}]^2 = p^2 + q^2$ (say)
Now applying Ns operation in between $(p^2 + q^2)$ & self we get the relation $17^3 + 2^7 = 71^2$.
We can proceed from the element 17 also. 17 must be expressed in the form of $p^2 - q^2$ where by successive N_d operations (3 times) on 17 we can get the same relation $17^3 + 2^7 = 71^2$, may be nature of p, q are different i.e. not in the form of p = s & $q = t\sqrt{u}$ where s, t, u are integers. For integer values of s, t, u N_d operations will produce relations of high value elements.

From Eq.(A) we can say $(p + q)^3 = 17 + 8\sqrt{2}$ & $(p - q)^3 = 17 - 8\sqrt{2}$
$\Rightarrow p = \frac{1}{2}.[(17 + 8\sqrt{2})^{1/3} + (17 - 8\sqrt{2})^{1/3}$ & $q = \frac{1}{2}.[(17 + 8\sqrt{2})^{1/3} - (17 - 8\sqrt{2})^{1/3}$

Computer generated some relations are given below.
$7^3 + 13^2 = 2^9$
$3^5 + 11^4 = 122^2$
$17^7 + 76271^3 = 21063928^2$
$1414^3 + 2213459^2 = 65^7$
$9262^3 + 15312283^2 = 113^7$
$43^8 + 96222^3 = 30042907^2$
$33^8 + 1549034^2 = 15613^3$ etc.
All can be explained in similar ways.

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III. Conclusion

The total nos. of solutions of $a^x + b^y = c^z$, where (a, b, c) is a prime set & any two of (x, y, z) > 2 & other = 2, seems to be finite. If it is so, how many? It needs further investigations.
Moreover, from N-eq. so many important things can be noticed such as:

a) if $(e_1^2 + o_1^2)(e_2^2 + o_2^2)$ produces a relation $e_3^2 + o_3^2 = e_4^2 + o_4^2$, then
$\mathrm{Max}(e_3, e_4) + \mathrm{Max}(o_3, o_4) = (e_1 + o_1)(e_2 + 0_2)$
$|\ \mathrm{Max}(e_3, e_4) - \mathrm{Max}(o_3, o_4)\ | = |(e_1 - o_1)(e_2 - 0_2)|$

b) if $(a_1^2 - b_1^2)(a_2^2 - b_2^2)$ produces a relation $a_3^2 - b_3^2 = a_4^2 - b_4^2$, then
$(a_1^2 + b_1^2)(a_2^2 + b_2^2)$ will produce a relation $a_3^2 + b_4^2 = a_4^2 + b_3^2$

c) For a N-equation $a^2 + b^2 = c^2$ where a & c are odd integers, the prime numbers excepting two can be divided into two types. Those who belong to 'c' as a prime factor or alone can be said as type-2 and the rest can be said as type-1.
Obviously, type-2 prime nos. are distributed to all values of 'k' and remains present for a particular value of k uniquely i.e. $k = c - \mathrm{Max}\{ 2ab, (b^2 - a^2)\}$ whereas type-1 & type-2 both are belonging to 'a' under k = 1 as a prime factors or along.

If P_2 be a prime no. of type-2, then $P_2^{2^n}$, n = 0, 1, 2, 3, ...; can be expressed as $a^2 + b^2$ uniquely
All the prime numbers of type-1 & type-2 both with exponent 2^n, n = 0, 1, 2, 3, ….. can be expressed as difference of two square quantities of two consecutive nos. uniquely i.e. $\{(P + 1)/2\}^2 - \{(P - 1)/2\}^2$. But for type-1 prime number it is $(\text{even})^2 - (\text{odd})^2$ & for type-2 prime no. it is $(\text{odd})^2 - (\text{even})^2$.
Any composite no. whose all prime factors are of type-2 can be expressed as $(e)^2 + (o)^2$ & $(o)^2 - (e)^2$ but not uniquely.
Any composite no. whose at least one prime factor is of type-1 can be expressed as $(e)^2 - (0)^2$ but not uniquely. It cannot be expressed as $(e)^2 + (o)^2$

When N is found to be prime by digital analysis we can establish the following fact.

Digit of unit Place of N	Digit of 10^{th} Place of N	Remarks
1 or 9	even	'N' is of type-2
3 or 7	odd	'N' is of type-2
1 or 9	odd	'N' is of type-1
3 or 7	even	'N' is of type-1

$\Rightarrow D(P_1 - 1) = 1$ & $D(P_2 - 1) > 1$ or, $(P_1 + 1)/2$ = even & $(P_2 + 1)/2$ = odd
* For any even number $N = 2^n.p$ where p is an odd integer, n is said to b degree of intensity & denoted by D.

So there lies ample of scopes for further development of N-eq. particularly in the field of prime numbers.

References

Books

[1] Academic text books of Algebra.

[2] In August edition, Vol-4 & issue 8 of IJSER, Author DEBAJIT DAS,